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Comment on "Paleoclassical Transport in Low-Collisionality Toroidal Plasmas" [Phys. Plasmas 12, 092512 (2005)]

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August 9, 2007

Physics of Plasmas

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August 3, 2007

Paleoclassical transport is a recently proposed fundamental process that is claimed to occur in current-carrying resistive plasmas and to be missing in the collisional drift-kinetic equations (DKE) in standard use. In this Comment we raise three puzzles presented by paleoclassical transport as developed in [1], one concerning conservation and two concerning uniqueness.

For convenient reference below, we highlight selected features of paleoclassical transport as developed in [1] (these statements are not a complete description of either paleoclassical processes or the magnetic configurations in which they occur):

S-I. Paleoclassical transport occurs in a strictly axisymmetric current-carrying resistive plasma in a torus.

S-II. If $\left(\frac{\partial\psi}{\partial t}\right)_{\mathbf{x}}$ goes to zero in [1] ($2\pi\psi$ is the poloidal flux), terms are absent in some equations but the calculations appear to go through straightforwardly; *i.e.*, without a structural change.

S-III. The paleoclassical electron thermal diffusivity $\chi_e^{\rm pc}$ depends only on the safety-factor q and local plasma profiles. There is no explicit dependence on the loop voltage, V_ℓ.

S-IV. The 6D kinetic equation—Vlasov operator plus Fokker-Planck collisions—is said to be correct and to contain paleoclassical transport.

S-V. Particles which are collisionless at least through the drift timescale diffuse with the magnetic flux.

In Statement S-V, we refer not to entire particle distributions responsible for the plasma resistivity, η (various collisionality regimes for the bulk electrons are considered in [1]), but, for example, to individual relatively high-energy electrons. The phenomenological derivation in [1, Sec. VI] concludes “that electron guiding centers are advected and diffused radially with the same Fokker-Planck coefficients as those for poloidal magnetic flux (field lines).” This is the key hypothesis of the paleoclassical model. No effect of a finite collision-time for the electron whose guiding-center is under consideration is invoked, and the derived $\chi_e^{\rm pc}$ is independent of the particular electron’s velocity. From this it is clear that paleoclassical transport is not a correction to the collision operator. Rather, paleoclassical transport is due to particles' guiding centers being nearly tied to ψ as it convects and diffuses in a current-carrying resistive plasma. It is the guiding-center motion in the standard DKE for such plasmas that is said to be in error.

Re S-I, note that the small helical distortions arising from the transport [1] are not necessary to cause the transport. (Helical resonances lead to a large multiplier on the axisymmetric result.)

From S-II, we are free to apply the model to configurations with vanishing inductive electric field. We therefore begin with the simplest case, restricting the discussion to 100% non-inductively driven resistive steady states (NISS); *i.e.*, to the case with static electric and magnetic fields.[4] The poloidal flux still satisfies a (steady-state) diffusion equation (the ∇^2 term is balanced by the source term (current drive)), and the expression given in [1] for $\chi_e^{\rm pc}$ is unaffected by the steady-state condition (S-III). In the appendix, we generalize slightly to consider Ohmic drive in axisymmetric steady states.

Paleoclassical puzzle 1: In an axisymmetric NISS plasma, the canonical angular momentum of a collisionless particle is conserved, so a collisionless particle is not free to diffuse with diffusing poloidal flux. If the timescale in S-V can be extended through the magnetic flux-diffusion timescale, there is a clear problem with conservation in the paleoclassical model. If it cannot be extended, two questions arise: (1) Where does a collision time (for the particle whose guiding-center orbit is under consideration) smaller than *vs.* greater than the magnetic flux-diffusion timescale enter in the derivation of Eq. (91) [1]? (2) The dissipation of a single electron's angular momentum in a NISS plasma—and so its rate of departure from a constant angular-momentum surface—depends only on the static $\mathbf{B}$ and $\mathbf{E}$ and the electron's own collision time. But in the paleoclassical model, the electron's paleoclassical motion off surfaces is additive and controlled entirely by the rate of magnetic flux-diffusion; it is independent of the particular electron's $\mathbf{v}$. How is this resolved?

Paleoclassical puzzle 2: Whether or not ψ obeys a diffusion equation, collisionless particle orbits depend only on $\mathbf{B}$ and $\mathbf{E}$. Consider now a NISS force-free plasma ($\beta \to 0,\ \mathbf{V} \to 0,\ \upsilon/\eta \to 0$, where β is the ratio of material-to-magnetic pressure, $\mathbf{V}$ the fluid velocity, and υ the viscosity). In this case, $\mathbf{B}$ depends only upon $\mathbf{J}_\parallel(\mathbf{x})$. Given flexibility in electron and ion heat- and particle-sources, one can construct solutions of the steady-state transport equations with different resistivity profiles but identical $\mathbf{E}$, while adjusting the current sources as needed in response to the density- and temperature-profile changes so that $\mathbf{J}_\parallel$ does not change. These solutions lead to different predictions for the rate of paleoclassical diffusion (which, again, depends only on field-line geometry and η). In a gyro-averaged description of the motion, the paleoclassical diffusion of guiding centers is in addition to the usual guiding-center drifts, which do not change as the resistivity changes. However, the full orbit is unique for given **B** and **E** for a collisionless particle; and it is unique in a statistical sense for given **B**, **E** and plasma profiles for a collisional particle. Identical equilibria with differing η thus present a puzzle—non-unique orbits—for any particle whose neoclassical and classical transport is small *cf.* the paleoclassical diffusion rate.

Paleoclassical puzzle 3 arises from the key hypothesis S-V itself and the related comments, "magnetic-field lines diffuse radially faster than collisions cause electrons to diffuse relative to them" [1, Sec. I], "The introduction of plasma resistivity leads to radial diffusion of magnetic field lines" [1, below Eq. (64)], "Paleoclassical transport will be caused by electrons. . . being nearly 'frozen to' and hence carried with the poloidal flux" [1, Sec. VI], and "The poloidal magnetic flux ψ and hence field lines move relative to the toroidal flux ψ_t" [1, below Eq. (36)]. As is well known, magnetic field-lines do not have a physical identity that survives from one instant to the next [2]. A velocity field $\mathbf{v}_{\text{f.l.}}$ can be ascribed to them for convenience [5], but this velocity is not a measurable quantity and in general (depending on boundary conditions) there is freedom in its choice. Even in ideal MHD (where $\mathbf{E}_{\|} = 0$ and the perpendicular fluid velocity equals the $\mathbf{E}\times\mathbf{B}$ drift velocity), a slip between the $\mathbf{E}\times\mathbf{B}$ drift and the field-lines can be included if desired (again, boundary conditions permitting). In Ref. [2], the constraints on the possible $\mathbf{v}_{\text{f.l.}}$ are given for flux-conserving or line-preserving (*i.e.*, a line initially a field-line remains a field-line. A flux-conserving $\mathbf{v}_{\text{f.l.}}$ will be line-preserving, but not necessarily *vice versa*) choices. The freedoms in each case are apparent. For a static **B**, $\mathbf{v}_{\text{f.l.}} = 0$ is a permissible but not unique flux-conserving choice. We emphasize that magnetic flux still diffuses radially in NISS plasmas, despite the fact that $(\partial\psi/\partial t)_{\mathbf{x}}$ and $(\partial\mathbf{B}/\partial t)_{\mathbf{x}}$ vanish. It is clear that (1) in NISS plasmas (and in general), magnetic flux and magnetic field-lines need not move together; and (2) in the general toroidal time-dependent current-carrying resistive case, there may be no permissible $\mathbf{v}_{\text{f.l.}}$ [2], [3]. On the other hand, in cases with broken magnetic surfaces (*e.g.*, stochastic fields in the vicinity of the X-point or islands in tokamaks), ψ itself is not a good coordinate, although a $\mathbf{v}_{\text{f.l.}}$ may be at least locally defined. Is the definition of the object to which electrons are tied in the paleoclassical model then problem-dependent? For a problem in which permissible flux-conserving $\mathbf{v}_{\text{f.l.}}$ exist and are not unique, what is the basis for selecting which among them to take for the paleoclassical hypothesis?

We conclude with a remark upon S-IV. If "paleoclassical transport" is taken to mean the response of a particle (guiding-center or full) to collisional processes which necessarily involve at least two other particles, a description of discrete-particle effects that goes beyond the fluctuationless two-particle effects contained in the 6D Fokker-Planck collisional kinetic equation would seem to be indicated.

Acknowledgments

This work was performed under the auspices of the U.S. Department of Energy by the University of California Lawrence Livermore National Laboratory under Contract No. W-7405-ENG-48.

Appendix

Here we extend the discussion to include a steady inductive electric field. We consider an idealized Ohmic transformer, leading at steady state to the time-independent Faraday's law $\nabla \times \mathbf{E} = -\bar{V}_\ell \delta(R)\hat{\mathbf{z}}/(\pi R)$, with $\bar{V}_\ell$ a constant. $(\partial \mathbf{E}/\partial t)_\mathbf{x}$ and, except at $R = 0$, $(\partial \mathbf{B}/\partial t)_\mathbf{x}$ vanish. Taking a path around any closed toroidal loop gives the toroidal loop-voltage $V_\ell \equiv -\oint d\ell \cdot \mathbf{E} = -2\pi R E_{\mathrm{tor}} = -2\pi\left(\frac{\partial\psi}{\partial t}\right)_\mathbf{x}$; note that the surface defining the poloidal flux ($2\pi\psi \equiv \int \mathbf{B}_{\mathrm{pol}} \cdot dS$) here is a disk (no holes) whose perimeter is a closed toroidal loop on the magnetic surface. For the given steady-state Faraday's law, $V_\ell = \bar{V}_\ell$. The total electric field at steady state is then $\mathbf{E} = -\nabla\phi - \bar{V}_\ell\hat{\varphi}/(2\pi R)$, where $\hat{\varphi}$ is the unit vector in toroidal angle and ϕ is the scalar potential, with $\partial\phi/\partial\varphi = 0$. The magnetic differential equation [5] in this case easily reduces to $\mathbf{B} \cdot \nabla(\phi + \bar{V}_\ell\varphi/(2\pi) - s) = 0$, or $\phi + \bar{V}_\ell\varphi/(2\pi) - s = g$, where ∇g is single-valued and normal to surfaces but otherwise arbitrary. There is no further constraint on $E_\|$, and puzzle 3 is essentially unchanged; in particular, $\mathbf{v}_{\mathrm{f.l.}} = 0$ is still a permissible and flux-conserving choice for the magnetic field-line velocity. Re puzzle 2, consider first an axisymmetric, fully Ohmically driven plasma. The surface-averaged Faraday's law in this case is:

$$\langle \mathbf{E} \cdot \mathbf{B} \rangle = -\frac{I}{2\pi}\left\langle \frac{V_\ell}{R^2} \right\rangle + \langle \mathbf{B}_{\mathrm{pol}} \cdot \mathbf{E} \rangle, \tag{1}$$

where $I \equiv RB_{\mathrm{tor}}$ and $\langle f \rangle \equiv \oint \frac{dl_{\mathrm{pol}}}{|\mathbf{B}_{\mathrm{pol}}|} f / \oint \frac{dl_{\mathrm{pol}}}{|\mathbf{B}_{\mathrm{pol}}|}$, the usual magnetic-surface average. Only the inductive $\mathbf{E}$ survives the surface averages. In steady-state, the toroidal flux is constant in time; then the last term in Eq. (1), proportional to the poloidal loop-voltage, vanishes. Noting that $I\langle 1/R^2 \rangle = \langle \mathbf{B} \cdot \nabla\varphi \rangle$, our steady-state flux-diffusion equation becomes:

$$-\frac{\bar{V}_\ell}{2\pi} = \left(\frac{\partial\psi}{\partial t}\right)_\mathbf{x} = \eta\frac{\langle \mathbf{J} \cdot \mathbf{B} \rangle}{\langle \mathbf{B} \cdot \nabla\varphi \rangle}. \tag{2}$$

(Writing $\mathbf{J}$ in terms of a second-order spatial derivative in the $\nabla\psi$-direction on ψ yields the "radial" diffusion equation for ψ.) Denote the values of the loop-voltage, resistivity, *etc.*, at the steady-state solution of Eq. (2) with a set of reference heat and particle sources by superscript "0".

A partially inductively driven plasma obeys the magnetic flux-diffusion equation

$$\eta\langle (\mathbf{J} - \mathbf{J}_{\mathrm{CD}}) \cdot \mathbf{B} \rangle = -\frac{I}{2\pi}\left\langle \frac{V_\ell}{R^2} \right\rangle + \langle \mathbf{B}_{\mathrm{pol}} \cdot \mathbf{E} \rangle. \tag{3}$$

At steady state, with driven current $\mathbf{J}_{\rm CD}$ adjusted so that $\langle \mathbf{J}\cdot\mathbf{B}\rangle/\langle \mathbf{B}\cdot\nabla\varphi\rangle = \langle \mathbf{J}^0\cdot\mathbf{B}^0\rangle/\langle \mathbf{B}^0\cdot\nabla\varphi\rangle$ (using this to determine II' while simultaneously solving the Grad-Shafranov equation guarantees that $\mathbf{J}=\mathbf{J}^0$), this reduces to

$$
\begin{aligned}
-\frac{\bar{V}_\ell}{2\pi} = \left(\frac{\partial\psi}{\partial t}\right)_{\mathbf{x}} &= \eta\frac{\langle(\mathbf{J}^0-\mathbf{J}_{\rm CD})\cdot\mathbf{B}^0\rangle}{\langle\mathbf{B}^0\cdot\nabla\varphi\rangle} \\
&= \eta\left[-\frac{\bar{V}_\ell^0}{2\pi\eta^0} - \frac{\langle\mathbf{J}_{\rm CD}\cdot\mathbf{B}^0\rangle}{\langle\mathbf{B}^0\cdot\nabla\varphi\rangle}\right].
\end{aligned}
$$

Next, we can choose a constant α and readjust the current-drive sources together with the heat sources so that $\langle \mathbf{J}_{\rm CD}\cdot\mathbf{B}\rangle/\langle \mathbf{B}\cdot\nabla\varphi\rangle = -\alpha\bar{V}_\ell^0/(2\pi\eta^0)$ while also maintaining (for given α and $\bar{V}_\ell$) η/η^0 in some constant ratio across the plasma in steady-state. Then

$$\bar{V}_\ell = \bar{V}_\ell^0\frac{\eta}{\eta^0}(1-\alpha). \tag{4}$$

Finally we can adjust $\mathbf{J}_{\rm CD}$ and the heat and particle sources together, so that at steady state, in addition to the conditions on $\mathbf{J}$ and η/η^0, we find $\phi=\phi^0$. If this last exercise is carried out for different α but the same $\bar{V}_\ell$ and reference configuration, the results will have identical $\mathbf{B}$ and $\mathbf{E}$ but different η, and puzzle 2—re uniqueness of orbits—follows as before. In light of Eq. (4), we revisit the general discussion of S-II at the beginning of this Comment. (Note that $\alpha=1 \Leftrightarrow \bar{V}_\ell=0$ is the electrostatic case, *i.e.*, a NISS plasma.) Does $\chi_e^{\rm pc}$ go to zero continuously as $\bar{V}_\ell$ goes to zero? If η/η^0 is held fixed while α is varied, the resulting steady-state plasmas will have identical $\mathbf{B}$, ϕ, and η—and therefore $\chi_e^{\rm pc}$—profiles. If $\chi_e^{\rm pc}$ goes to zero continuously, and if $\chi_e^{\rm pc}=0$ for a NISS plasma, how is this compatible with S-III? Finally we remark on the development of Eq. (3) to obtain the magnetic-flux-diffusion equation for the general axisymmetric time-dependent case: Had we proceeded (transforming from $\mathbf{x}$ to a radial coordinate, *etc.*, as in [1] and references therein), and then imposed the steady-state condition, the results here would be the same. This is true independent of the choice of radial coordinate [6].

Re puzzle 1, particle orbits are affected by the constant $\left(\frac{\partial\psi}{\partial t}\right)_{\mathbf{x}}$: there is now an inward pinch from the inductive $\mathbf{E}\times\mathbf{B}$ drift $c\bar{V}_\ell\nabla\psi/(2\pi R^2B^2)$. The angular momentum of collisionless particles is still conserved, however, and they cannot diffuse or convect away from the constant angular-momentum surfaces, which now shrink as ψ changes with the constant $\left(\frac{\partial\psi}{\partial t}\right)_{\mathbf{x}}$. (To maintain this configuration in steady state will require sources and sinks for the collisionless particles. We choose to restrict these to small volumes localized around the edge and the magnetic axis. Paleoclassical puzzle 1 for such particles is then applicable to the source-free region in between.) Similarly for the weakly collisional particles: the addition of the inward pinch does not affect the argument in the main text.

[1] J. D. Callen, Phys. Plasmas **12**, 092512 (2005).

[2] W. A. Newcomb, Ann Phys. **3**, 347 (1958). A recent discussion of the existence and uniqueness of magnetic-field-line velocities, including a model time-dependent axisymmetric toroidal example, can be found in A. L. Wilmot-Smith, E. R. Priest, and G. Hornig, Geo. and Astrophys. Fluid Dyn. **99**, 177 (2005). A brief review including select-but-key references is given in D. H. Nickeler and H.-J. Fahr, Solar Physics **235**, 191 (2006).

[3] W. A. Newcomb, Phys. Fluids **2**, 362 (1959).

[4] Strict steady state is not required for the considerations here, merely steady state on the resistive-diffusion timescale. In principle, particle-, heat- and current-drive sources can be arranged to accomplish this. Current drive is discussed in [1] below Eq. (67). If the paleoclassical model cannot be applied as $\left(\frac{\partial\psi}{\partial t}\right)_{\mathbf{x}}$ becomes small, then the questions are: In the derivation of χ_e^{pc} in [1], where does the problem occur as $\left(\frac{\partial\psi}{\partial t}\right)_{\mathbf{x}}$ becomes small? Does χ_e^{pc} go to zero continuously as $\left(\frac{\partial\psi}{\partial t}\right)_{\mathbf{x}}$ goes to zero?

[5] This assumes that the magnetic differential equation $\mathbf{B}\cdot\mathbf{E} = -\mathbf{B}\cdot\nabla s$ can be solved for a single-valued ∇s [3]—always the case for an electrostatic $\mathbf{E}$.

[6] Often, before transforming from $\mathbf{x}$ to a flux-surface coordinate, one first takes the derivative with respect to ψ_t of the flux-diffusion equation Eq. (2) or (3), arriving at a diffusion-equation for q. (To the author's knowledge, S. Jardin was the first to point out that solving the transport equation for q rather than for a magnetic flux eases the numerics in coupling to a Grad-Shafranov solver. It eliminates having to take a numerical derivative.) One then transforms the independent variable of this diffusion-equation from $\mathbf{x}$ to a flux-surface-labeling coordinate (other than q, of course), typically chosen as some function of normalized poloidal or toroidal flux for convenience—*e.g.*, to reduce the numerical error in computational applications, or, in the case of RFP's, to avoid a double-valued problem were ψ_t to be used as the independent variable. Note that the choice of ψ_t for the independent flux-coordinate in [1], discussed above Eq. (1) in the paper, while well-motivated for tokamaks, does not introduce an error related to the aspect-ratio in the flux-diffusion equation itself; this choice still takes $\left(\frac{\partial\psi_t}{\partial t}\right)_{\mathbf{x}}$, which does not vanish in tokamaks (except in resistive steady state), fully into account.

This work was performed under the auspices of the U.S. Department of Energy by
University of California, Lawrence Livermore National Laboratory under
Contract W-7405-Eng-48.